FEDESARROLLO

Fundación para la Educación Superior y el Desarrollo

The Technical Specification of FEDESARROLLO´s
Long Run General Equilibrium Model

Maurizio Bussolo
David Roland - Holst
Dominique van der Mensbrugghe

The Technical Specification of FEDESARROLLO's Long Run General Equilibrium Model[1]

Maurizio Bussolo[2]
Fedesarrollo, A.A. 75074, Bogotá - Colombia
and Universidad Los Andes

David Roland-Holst
OECD Development Centre,
Mills College, and CEPR

Dominique van der Mensbrugghe
OECD Development Centre,

ABSTRACT

This technical paper, prepared as part of FEDESARROLLO's research programme on Colombian economic policy analysis, presents the complete technical specification of the current version of the COGEM (*CO*lombian *G*eneral *E*quilibrium *M*odel). The document lists all the model equations and by explaining details of the various specifications, which are normally bypassed in technical papers, gives full justification of the modelling choices. This paper, together with a companion document on the model database, are intended to inform in the most detailed way researchers interested in replicating or expanding FEDESARROLLO's modelling effort, and to provide a useful instrument in informing the debate on economic policy in Colombia.

1 Introduction

Empirical modelling in support of economic policy has a long history, extending at least as far back as the seventeenth century and Quesnay's Tableau Economique. Calibrated general equilibrium (CGE) models represent the most recent and in many ways the most ambitious of our attempts to simulate the workings of the economy in a way which yields practical information for policy makers. CGE models distinguish themselves from their predecessors in attempting a coherent empirical synthesis of modern neoclassical microeconomics at an economywide scale. This technical paper, prepared as part of FEDESARROLLO's research programme on Colombian economic policy analysis, presents the complete technical specification of the current version of its CGE model, and it also adds a simple illustrative simulation example.

Readers interested in broader introductions to applied general equilibrium methodology can consult some of these surveys: Borges (1986), Dervis, de Melo, and Robinson (1982), and Whalley (1986). Trade policy applications of CGE models are surveyed in de Melo (1988), Shoven and Whalley (1984), and Whalley (1989b). The extensive literature on less developed country applications is surveyed in de Melo (1988), Devarajan (1987), Devarajan, Lewis, and Robinson (1986), and Robinson (1988,1989). Hertel (1989) surveys applications to agriculture. The use of these models with specifications of imperfect competition is surveyed by Richardson (1989), but this is the fastest area of growth and change, and a number of important references cited here are better consulted directly.[3] Finally, the technical side of these models, such as efficient solution algorithms, is also under intensive interdisciplinary research, and these efforts are cited in Harris (1988).

The paper is organised as follows. The next section provides a compact overview of the main characteristics and dimensions of the model. Section 3 describes the algebraic structure of the model block-by-block, explaining the motivation for the chosen specifications. Section 4 illustrates an example of an application of the model to policy analysis; in particular the effects of a reduction in government expenditure are assessed.

2 Overview of the model

The model is calibrated on data contained in the Social Accounting Matrix estimated for the year 1994 and described in detail in Bussolo, Correa and Prada (1998). The version of the SAM currently used here includes 10 household categories (5 urban and 5 rural), 23 sectors, 3 labour types, 3 separated trading partners.[4] The model is dynamic and solved recursively for the years 1994 through 2006. It includes approximately 100 generic equations describing agent behaviour, market clearing and other accounting relationships. The following sub-sections briefly illustrate the model's main characteristics.

---

[3] de Melo and Tarr (1989) is the most prominent example.

[4] A more sectorally disaggregated SAM could have been derived from the original SAM and used to calibrate the model. The 23-sector SAM though combines enough detail with reasonable tractability.

2

Production

The Constant Elasticity of Substitution (CES) constant returns to scale production function is a nested structure taking into account the assumed substitution possibilities in the choice of production factors. Output results from two composite goods: non-energy intermediates and energy plus value added. The intermediate aggregate is obtained combining all products in fixed proportions (Leontief structure). The value added and energy components are decomposed in two parts: aggregate labour and capital, which includes energy. Labour is a composite of 3 categories. The capital-energy bundle is further disaggregated into its basic components.[5] By distinguishing between "new" and "old" vintages, the capital existing at the beginning of each period, or already installed, can be separated from that resulting from contemporary investment (putty/semi-putty production function).[6]

Finally, the energy aggregate includes two energy substitutes: oil and electricity. Figure 1-1 depicts the nested decision process in the choice of production factors.

Substitution elasticities reflect adjustment possibilities in the demand for factors of production originating from variations in their relative prices. Consider particular values[7]: 0.00 between intermediates and value added with *old* capital plus energy; 0.50 between intermediates and value added aggregate incorporating *new* capital plus energy; 0.12 between aggregate labour and *old* capital-energy bundle; 1.00 between aggregate labour and *new* capital-energy bundle; 0.40 among different types of labour; 0.00 between *old* capital and energy; 0.80 between *new* capital and energy; 0.25 among different sources of energy associated with *old* capital; 2.00 among those associated with *new* capital.

Income Distribution and Absorption

Labour income is allocated to households according to a fixed coefficient distribution matrix derived from the original SAM. Likewise capital revenues are distributed among households, corporations and rest of the world. Corporations save the after-tax residual of that revenue.
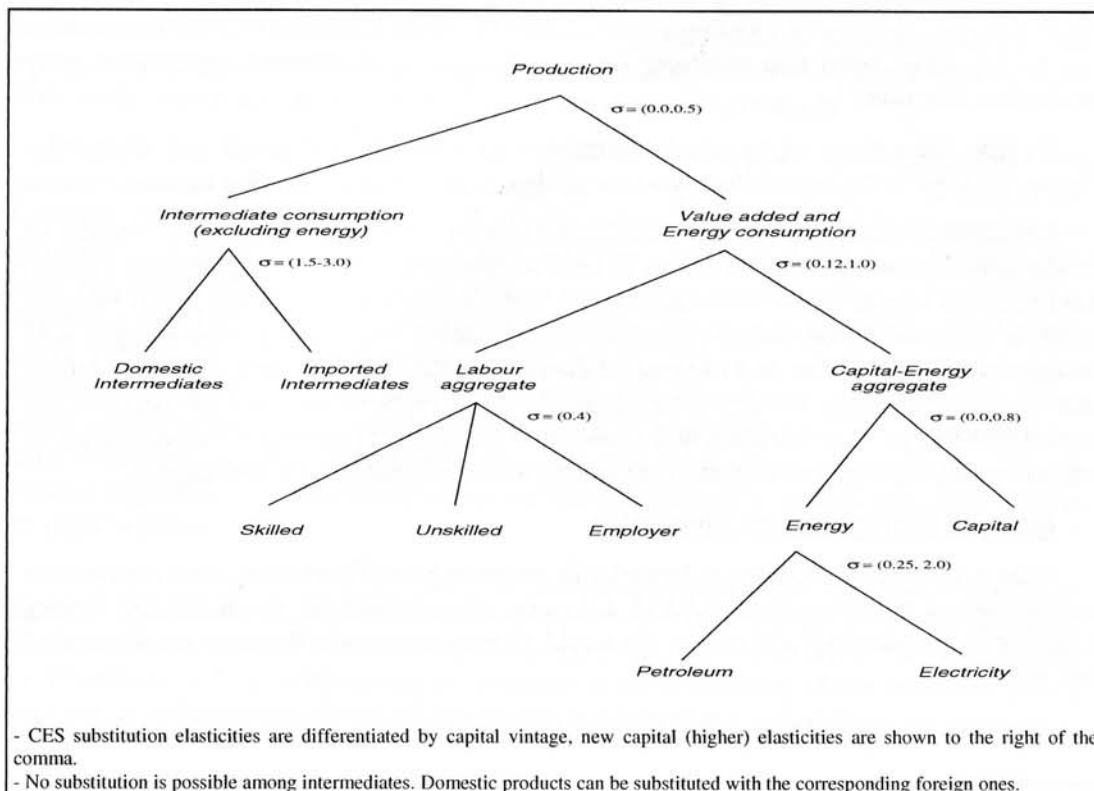
---

[5] The particular production function of this model treats energy as a separate factor of production rather than an intermediate input. Energy use is typically highly polluting and the specific nesting structure adopted here allows monitoring more closely energy-related emissions (for instance in a study of the environment). Moreover bundling energy together with capital is motivated by the fact that new technologies, embodied in new capital goods, are usually energy saving (i.e. energy substituting).

[6] In the short run capital is usually sector-specific, whereas in the long run it can be perfectly mobile across sectors. The "vintages" approach allows integrating in the present dynamic model both short run capital immobility and long run capital mobility. In the modelled economy new capital (equal to the previous period's level of investment) is perfectly mobile and old capital only partially mobile across sectors. Another advantage of the "vintages" approach is that it allows introducing different degrees of substitutability of capital with other factors. In fact, old capital vintage is less substitutable with energy, labour and other inputs than new capital. Both these features add realism to this model where enhanced openness should increase investment opportunities and new capital goods should embody cleaner technologies and greater adjustment possibilities.

[7] These elasticities are derived from the most recent relevant literature. In fact, they are mostly derived from background studies done for the construction of the OECD GREEN model. See for instance Burniaux, Nicoletti and Oliveira-Martins (1992).

3

Private consumption demand is obtained through maximisation of household specific utility function following the Extended Linear Expenditure System (ELES).[8] Household utility is a function of consumption of different goods and saving. Income elasticities are different for each household and product and vary in the range 0.20, for basic products consumed by the household with highest income, to 1.30 for services.[9] Once their total value is determined, government and investment demands[10] are disaggregated in sectoral demands according to fixed coefficient functions.

Figure 1-1: Nested Production Function

- CES substitution elasticities are differentiated by capital vintage, new capital (higher) elasticities are shown to the right of the comma.
- No substitution is possible among intermediates. Domestic products can be substituted with the corresponding foreign ones.

International Trade

In the model we assume imperfect substitution among goods originating in different geographical areas.[11] Imports demand results from a CES aggregation function of domestic and imported goods. Export supply is symmetrically modelled as a Constant Elasticity of Transformation (CET) function. Producers decide to allocate their output to

[8] A useful reference for the ELES approach is found in Lluch (1973). More detailed explanations of this modelling choice are given in the following chapter.
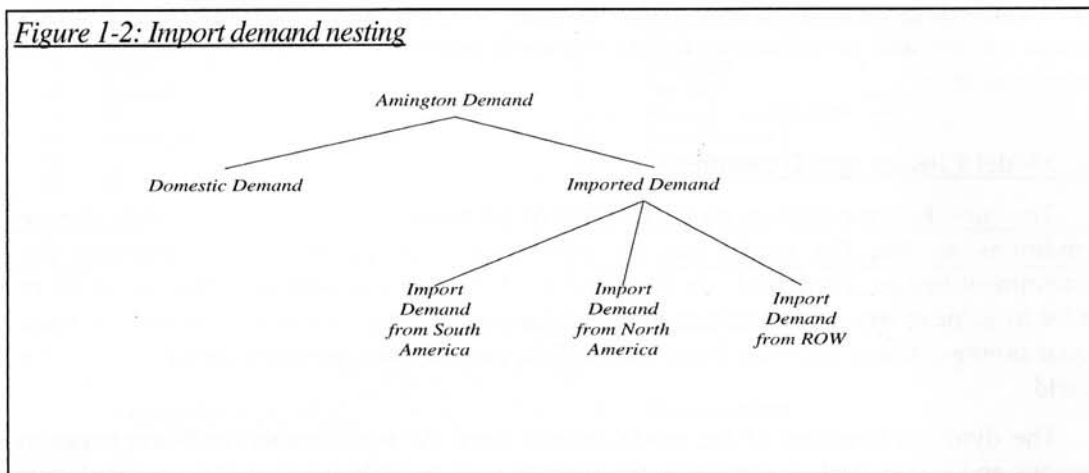
[9] Among the various sources for these elasticities see Blanciforti and Green (1983), Eastwood and Craven (1981), Lopez (1989) and Maki (1988).

[10] Aggregate investment is set equal to aggregate savings, while aggregate government expenditures are exogenously fixed.
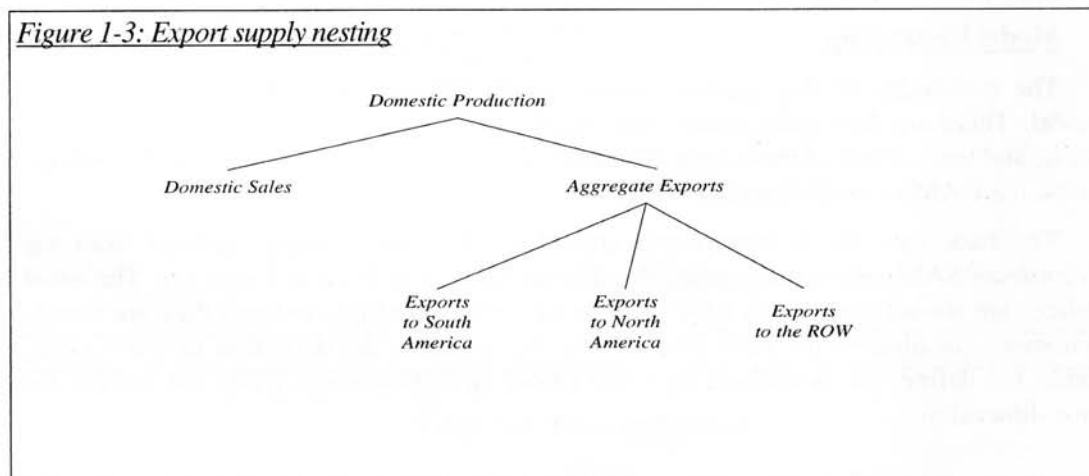
[11] Armington (1969).

4

domestic or foreign markets responding to relative prices. The model implements a two-stage procedure for determining both import demand and export supply. For imports consider Figure 1-2. At the first stage aggregate demand is decomposed into a domestic component and an aggregate import component. At the second stage, aggregate import demand is allocated across the various trading partners.

Figure 1-2: Import demand nesting

Amington Demand

Domestic Demand   Imported Demand

Import Demand from South America   Import Demand from North America   Import Demand from ROW

Export supply is treated in a symmetric fashion (see Figure 1-3). Producers allocate production between domestic sales and aggregate export sales. At the second stage, aggregate exports are sold to the various trading partners based on the relative price the exporter can receive in each market.[12]

Figure 1-3: Export supply nesting

Domestic Production

Domestic Sales   Aggregate Exports

Exports to South America   Exports to North America   Exports to the ROW

---

[12] Elasticities between domestic and foreign products are of comparable magnitude for imports demand and exports supply. Their values are 3.00 for agricultural goods, 2.00 for manufactured goods and 1.50 for services. Similar values are used for the second nesting.

5

As Colombia is unable to influence world prices the small country assumption holds, and its imports and exports prices are treated as exogenous. The balance of payments equilibrium is determined by the equality of foreign savings (which are exogenous) to the value for the current account. With fixed world prices and capital inflows, all adjustments are accommodated by changes in the real exchange rate: increased import demand, due to trade liberalisation must be financed by increased exports, and these can expand owing to the improved resource allocation. Price decreases in importables drive resources towards export sectors and contribute to falling domestic resource costs (or real exchange rate depreciation).

Model Closure and Dynamics

The equilibrium condition on the balance of payments is combined with other closure conditions so that the model can be solved for each period. Firstly consider the government budget. Its deficit[13] is fixed and the household income tax schedule shifts in order to achieve the predetermined net government position. Secondly, investment must equal savings, which originate from households, corporations, government and rest of the world.

The dynamic structure of the model results from the equilibrium condition between savings and investment. A change in the savings volume influences capital accumulation in the following period. Exogenously determined growth rates are assumed for various other factors that affect the growth path of the economy, such as: population and labour supply growth rates, labour and capital productivity growth rates and energy efficiency factor growth rate. Agents are assumed to be myopic and to base their decisions on static expectations about prices and quantities. The model dynamics are therefore recursive, generating a sequence of static equilibria.[14]

Model Dimensions

The remainder of this section introduces the dimensions of FEDESARROLLO's model. There are four main dimensions: sectors, regions, household's types and labour skills, and time. Some of these broad dimensions are split into sub-dimensions (or subsets to use the GAMS terminology).

The base data set is constructed around a 23-sector database, derived from the Colombian SAM mentioned previously. The sectors are defined in Table 1-1. The usual indices are shown under each table title. In the case of multiple indices, they are simply synonyms (or aliases) for each other. Table 1-2 provides the definition of the regions. Table 1-3 defines the household type and labour skill dimension. Table 1-4 defines the time dimension.

---

[13] Its initial value is determined in the 1994 SAM.
[14] The model's long-run properties are discussed in the next section.

Table 1-1: **Sectoral Definition**

(i,j)

1.	Agricultural food	13.	Other mining
2.	Cereal	14.	Processed food
3.	Oil seeds	15.	Consumption goods
4.	Other agricultural products	16.	Intemediate goods
5.	Pergamino coffee	17.	Metal products incl. machinery and equipment
6.	Livestock	18.	Construction
7.	Forestry, fishing	19.	Commerce
8.	Processed coffee	20.	Transport
9.	Oil	21.	Electricity, gas and water
10.	Natural gas	22.	Financial sector
11.	Coal	23.	Other services
12.	Oil refinery by products		

Table 1-2: **Regional Definition**

(r,r')

1.	North America	3.	Rest of the World
2.	South America		

Table 1-3: **Household types and Labour Skills**

(h)

1. Propietor
2. Informal Own Account
3. Informal Employer
4. Professional
5. Labourer
6. Goverment Household
7. Agricultural Household

(l)

1. Unskilled labour
2. Skilled labour
3. Other labour (independent workers, employers)

Table 1-4: **Time Definition**

(t)

1.	1994	7.	2001
2.	1995	8.	2002
3.	1996	9.	2003
4.	1997	10.	2004
5.	1998	11.	2005
6.	1999	12.	2006

3 Model blocks

3.1 Household Consumption

In many CGE models household expenditure behaviour functions are derived from the maximisation of Cobb-Douglas or Constant Elasticity of Substitution (CES) utility. The limitation of using these functional forms for consumption is that they imply unitary income elasticity of demand. This fails to account for the way changes in income affect the structural adjustment of the economy to exogenous shocks. In order to avoid such drawbacks, consumption demand in the current model is determined by using the utility function associated with the extended linear expenditure system (ELES). The ELES is similar to the LES or Stone-Geary system[15], but incorporates household saving into the utility function.

Consumers under the ELES are assumed to maximise the following utility function:[16]

$$\max U = \sum_i \mu_i \ln(C_i - \theta_i) + \mu_s \ln\left(\frac{S}{P}\right)$$

subject to the budget constraint:

$$\sum_i P_i^C C_i + S = Y^d$$

C is consumer spending, S is saving (in value), Y^d is disposable income, P^C are consumer prices, and μ and θ are the ELES parameters.[17] The Engel aggregation condition[18] requires the following constraints on the parameters μ:

$$\sum_i \mu_i + \mu_s = 1$$

The following demand functions can be derived:

$$C_i = \theta_i + \frac{\mu_i}{P_i^C}\left(Y^d - \sum_j P_j^C \theta_j\right)$$

The usual interpretation of this demand function is that consumption is composed of two parts. The first part has been referred to as the subsistence minima (or floor

---

[15] See Stone (1954).

[16] Note that the same specification has been used for each household type.

[17] In the utility function, S needs to be deflated by an appropriate price, which would represent the consumer spot price of future consumption. This price does not need to be specified for the model since household saving can be derived as a residual from the budget constraint. For welfare calculations, the consumer price index, *cpi*, has been chosen as the saving deflator since there is no forward-looking behaviour in EMMA.

[18] See Deaton and Muellbauer (1980) page 16.

consumption), θ .The term in parenthesis represents residual income, or *supernumerary income*, i.e. it is the residual income after subtracting expenditures on the subsistence minima. Therefore the second part of consumption is a share of supernumerary income. Note that there is no minimal consumption of savings, i.e. θ_s is 0 Saving can be determined via the budget constraint:

$$S = Y^d - \sum_i P_i^C C_i$$

The income and price elasticities are given by the following formula:

$$\eta_i = \frac{\mu_i Y^d}{P_i^C C_i} = \frac{\mu_i}{\chi_i}$$

$$\varepsilon_i = \frac{\theta_i(1-\mu_i)}{C_i} - 1$$

The income elasticity is equal to the ratio of the marginal propensity to consume good i out of supernumerary income, μ , over the average propensity to consume good i out of income.

The relevant model equations are presented in Table 2-5. Equation (2-5.1) defines supernumerary income. The subsistence minima are calibrated in the base year on a per capita basis, therefore they are multiplied each period by the total population (pop) in order to grow with population. The indices i and h identify the consumer goods and household type respectively. Equation (2-5.2) defines consumer demand in terms of Armington composite, X_{ih}^{AC}. Equations (2-5.3) and (2-5.4) define specific (S_h^H) and total (S_{tot}^H) household saving.

Table 2-5: **Household Consumption**

(2-5.1) $\quad Y_h^* = Y_h^d - Pop_h \sum_i P_{ih}^C \theta_{ih}$

(2-5.2) $\quad X_{ih}^{AC} = \theta_{ih} Pop_h + \frac{\mu_{ih}}{P_{ih}^C} Y_h^*$

(2-5.3) $\quad S_h^H = Y_h^d - \sum_i P_{ih}^C C_{ih}$

(2-5.4) $\quad S_{Tot}^H = \sum_h S_h^H$

Table 2-6 describes consumer prices. Equation (2-6.1) simply sets the consumer price equal to the Armington price. Equation (2-6.2) defines the consumer price index.

9

Table 2-6: **Consumer Prices**

(2-6.1) $\quad P_{ih}^C = PA_i$

(2-6.2) $\quad P_h^{CPI} = \dfrac{\sum\limits_i P_{ih}^C X_{ih}^{AC}}{\sum\limits_i P_{ih,0}^C X_{ih}^{AC}}$

3.2 *Other Final Demands*

Apart from household consumption, final demands include government current and capital expenditures, and private capital expenditures (private investment). These are integrated into a single final demand matrix component. All these final demand vectors are assumed to have fixed expenditure shares.

Equation (2-7.1) determines the composition of final demand components. The demands for goods are determined as constant shares of the volume of total final demand D^{TFD}. The index f covers government current and capital expenditures ($f=g$), and private investment ($f=inv$). Equation (2-7.2) determines the value of final demand expenditures, D^{VTFD}. Equation (2-7.3) determines the price of final demand expenditures, which, without any taxes or subsidies, is equal to the Armington price.

Table 2-7: **Final Demand Expenditure Equations**

(2-7.1) $\quad X_{if}^{AF} = a_{if}^{FD} D_f^{TFD}$

(2-7.2) $\quad D_f^{VTFD} = \sum\limits_i P_{if}^{FD} X_{if}^{AC}$

(2-7.3) $\quad P_{if}^{FD} = PA_i$

Government aggregate expenditures on goods and services (D_g^{TFD}) are fixed in real terms. Total nominal government expenditures, G^{Exp}, is determined in Equation (2-8.1) as the sum of total value of expenditures on goods and services (D_g^{VTFD} as in Equation (2-7.2)) plus two exogenous elements: transfers to the rest of the world, F^{Gout}, and transfers to households, T_h^G. Equation (2-8.2) defines the government expenditure deflator, P^G.

Table 2-8: **Government Expenditure Equations**

$$(2\text{-}8.1) \quad G^{Exp} = D_g^{VTFD} + E^{RATE} \sum_r F_r^{Gout} + \sum_h P^{Index} T_h^G$$

$$(2\text{-}8.2) \quad P^G D_g^{TFD} = \sum_i P_{ig}^{FD} X_{ig}^{AC}$$

3.3 Production

The production inputs choice is modelled as a nested structure with different degrees of elasticity of substitution (CES) at the different levels.

At the top level, the producer chooses a mix of value added aggregate (V^A) and an intermediate demand aggregate (N^D). The optimisation problem takes the following form:

$$\min P_i^V V_i^A + P_i^N N_i^D$$

subject to the production function:

$$XP_i = \left[a_i^V V_i^{A^{\rho_{i,p}}} + a_i^N N_i^{D^{\rho_{i,p}}} \right]^{1/\rho_{i,p}}$$

where P_i^V is the aggregate price of value added, P_i^N is the price of the intermediate aggregate, a_i^V and a_i^N are the CES share parameters, and ρ is the CES exponent. The exponent and the CES elasticity are related via this relationship:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma - 1}{\sigma}$$

note that in the model, the share parameters incorporate the substitution elasticity using the following relationships:

$$\alpha_i^V = \left(a_i^V \right)^\sigma \quad \text{and} \quad \alpha_i^N = \left(a_i^N \right)^\sigma$$

The solution to this minimisation problem yields Equations (2-9.1) and (2-9.3) in Table 2-9. Notice that because of the existence of vintage capital, each producing sector is modelled as comprising two distinct technologies, producing a homogeneous good, but with different production parameters. Hence, intermediate and value added aggregate demands are indexed by vintage (using the index v). Moreover, due to the importance of energy in terms of pollution, the demand for energy has been separated from the rest of intermediate demand, and incorporated in the value added nest. Hence, the equations below are not specified in terms of a value-added bundle, but a value added plus energy bundle. Equation (2-9.1) determines the volume of aggregate intermediate non-energy demand, by vintage, Nv^D. Equation (2-9.2) determines the total demand for non-energy intermediate inputs (summed over vintages), N^D. Equation (2-9.3) determines the level of the composite bundle of value added demand and energy Q^{KEL}.

Table 2-9: **Top Level Production Nest**

$$(2\text{-}9.1) \quad N_{jv}^{D} = \alpha_{jv}^{N} \left(\frac{PXv_{jv}}{P_{j}^{N}} \right)^{\sigma_{jv}^{P}} XPv_{jv}$$

$$(2\text{-}9.2) \quad N_{j}^{D} = \sum_{v} Nv_{jv}^{D}$$

$$(2\text{-}9.3) \quad Q_{jv}^{KEL} = \alpha_{jv}^{KEL} \left(\frac{PXv_{jv}}{P_{jv}^{KEL}} \right)^{\sigma_{jv}^{P}} XPv_{jv}$$

The next level of the production nest concerns on one side aggregate intermediate demand N^{D}, and, on the other side, the Q^{KEL} bundle. The relevant equations are shown in Table 2-10. In Equation (2-10.1), N^{D} is split in its single inputs components (at the Armington level, i.e. before disaggregation into import demand and demand for domestically produced commodities) assuming a Leontief technology. The index nf identifies elements pertaining to the set of non-energy commodities. Notice that in Equation (2-10.1) aggregate intermediate demand is determined directly (i.e. summing over vintage), since non-energy intermediate demand is not dependent on the vintage. The matrix a, is the matrix of input-output coefficients for non-energy intermediate inputs.

At the same level, the Q^{KEL} bundle is split into aggregate labour demand on the one hand L^{A}, and the Q^{KE} bundle on the other. This is done using a CES function with the substitution elasticity σ^{kel}, which is assumed to be vintage specific. Equations (2-10.2) and (2-10.3) provide the reduced form first order conditions for this level of the nest. The decomposition of aggregate labour demand into labour demand by skill type is independent of vintage therefore it is summed directly in Equation (2-10.2) where L^{Ad} represents aggregate sectoral labour demand. P^{KEL} is the aggregate (or CES dual) price of the Q^{KEL} bundle, W^{A} is the price of aggregate labour in each sector, and P^{KE} is the price of the Q^{KE} bundle. The share parameters are α^{L} for labour, and α^{KE} for the Q^{KE} bundle.

Table 2-10: **Second Level CES Production Equations**

$$(2\text{-}10.1) \qquad X_{nf,j}^{AP} = a_{nf,j} N_{j}^{D}$$

$$(2\text{-}10.2) \qquad L_{j}^{A^{d}} = \sum_{v} \alpha_{jv}^{L} Q_{jv}^{KEL} \left(\frac{P_{jv}^{KEL}}{W_{j}^{A}} \right)^{\sigma_{jv}^{KEL}}$$

$$(2\text{-}10.3) \qquad Q_{jv}^{KE} = \alpha_{jv}^{KE} Q_{jv}^{KEL} \left(\frac{P_{jv}^{KEL}}{P_{jv}^{KE}} \right)^{\sigma_{jv}^{KEL}}$$

The next level of the CES nesting disaggregates the Q^{KE} bundle into the energy bundle on one side, and capital demand on the other side. The equations in Table 2-11 provide the reduced form first order conditions for demand for E^P and Kv.

Table 2-11: **Demand for the Energy Bundle and Capital**

$$(2\text{-}11.1) \qquad E_{jv}^{P} = \alpha_{jv}^{E} Q_{jv}^{KE} \left(\frac{P_{jv}^{KE}}{P_{jv}^{EP}} \right)^{\sigma_{jv}^{KE}}$$

$$(2\text{-}11.2) \qquad Kv_{jv}^{d} = \alpha_{jv}^{K} \frac{Q_{jv}^{KE}}{\lambda_{jv}^{K}} \left(\frac{\lambda_{jv}^{K} P_{jv}^{KE}}{R_{jv}} \right)^{\rho_{k,j}^{k,v}}$$

$$(2\text{-}11.3) \qquad K_{j}^{d} = \sum_{v} Kv_{jv}^{d}$$

E^P is demand for the energy bundle (by vintage), P^{Ep} is the price of the energy bundle, Kv^d represents capital demand by vintage, and R is the vintage specific rental rate of capital. The share parameters are α^E for the energy bundle, and α^K for capital. Capital demand incorporates changes in capital factor efficiency. Equation (2-11.3) determines aggregate sectoral capital demand.

There remain two more bundles to decompose: aggregate labour and the energy bundle. Table 2-12 lists the equations for determining labour demand by skill type and energy demand by fuel type.

There is a single nesting for labour by type, which implies that the substitution between any pair of labour skills is the same. L^d represents demand for labour of type l in sector j, the CES share parameters are α^{LS}, and the CES substitution elasticity for labour is σ^L. Labour is assumed to be perfectly mobile across sectors which implies a uniform economy-wide wage rate. However, we allow for the possibility of differential sectoral wages (for the same labour skill) to take into account observed data which reflect specific institutional features. The parameter ω is fixed and determines the relative wage across sectors, where W represents the equilibrium wage for skill l. Finally, the parameter λ^L incorporates labour efficiency improvement.

Table 2-12: **Demand for Labour by Skill Type and Energy by Fuel Type**

$$(2\text{-}12.1) \qquad L_{lj}^{d} = \alpha_{lj}^{LS} \frac{L_{j}^{A^{d}}}{\lambda_{lj}^{L}} \left(\frac{\lambda_{lj}^{L} W_{j}^{A}}{\omega_{lj} W_{l}} \right)^{\sigma_{j}^{L}}$$

$$(2\text{-}12.2) \qquad X_{e,j}^{AP} = \sum_{v} \alpha_{e,j,v}^{EP} \frac{E_{jv}^{P}}{\lambda_{jv}^{EP}} \left(\frac{\lambda_{jv}^{EP} P_{jv}^{EP}}{PA_{e}} \right)^{\sigma_{jv}^{EP}}$$

13

Energy demand is vintage specific, and the substitution possibilities across fuels are generally lower for old capital than for new capital. The current version of the model uses a single energy nesting, i.e. the decomposition of the energy bundle into the fuel components requires only one CES function. The index e represents the fuel commodities in the sectoral disaggregation. Equation (2-12.2) determines the demand for each fuel and incorporates energy efficiency improvement which is both sector and vintage specific (but not fuel specific).

This completes the description of the production structure. Starting from output, XPv, the nested CES tree structure of production unfolds until at the end of each branch a basic commodity (at the Armington level) or factor of production is specified. The next section will describe the formulation of prices in the production sector. The description of prices proceeds in the opposite direction. It starts at the bottom of the tree, using the fundamental (or the economy's equilibrium prices), and moves up the tree to define the price of the different CES aggregate bundles.

A graphical description of the production structure is given in Figure 1-1 in the previous chapter.

3.4 Production Prices

In this section it is assumed that all equilibrium prices are given. The equilibrium prices include the Armington prices and the factor prices. The aggregate prices are all determined going from the bottom up. Table 2-13 describes the (CES) price of the energy bundle. It is an aggregation of the Armington price of the individual fuels.

Table 2-13: **Price of the Energy Bundle in Production**

$$(2\text{-}13.1) \qquad P_{jv}^{EP} = \left[\sum_{@} \alpha_{e,j,v}^{EP} \left(\frac{PA_e}{\lambda_{jv}^{EP}} \right)^{1-\sigma_{yj}^{EP}} \right]^{1/(1-\sigma_{yj}^{EP})}$$

Similarly, Equation (2-14.1) defines the aggregate price of labour by sector. It is the CES dual price of the skill-specific wages.

Table 2-14: **Aggregate Price of Labour**

$$(2\text{-}14.1) \qquad W_j^A = \left[\sum_l \alpha_{lj}^{LS} \left(\frac{\omega_{li} W_l}{\lambda_{li}^L} \right)^{1-\sigma_j^L} \right]^{1/(1-\sigma_j^L)}$$

Table 2-15 provides the equations describing the remaining prices in production. The price of aggregate non-energy intermediate demand, specified in Equation (2-15.1), is given by adding up the unit price of non-energy input goods. Equation (2-15.2) determines the CES dual price of the capital-energy bundle, PKE. The price of the Q^{KEL} bundle is provided by the formula in Equation (2-15.3). Equation (2-15.4) determines the CES dual price of production by capital vintage, PXv. Equation (2-15.5) determines the average unit cost of production, PX, averaged over both types of capital. Finally, Equation (2-15.6) provides the producer price, PP, which is equal to the cost of production plus an indirect tax.

Table 2-15: Price of the Q^{KE} and Q^{KEL} Bundles, and Unit Production Cost

(2-15.1)
$$P_j^N = \sum_{nf} a_{nf,j}^N PA_{nf}$$

(2-15.2)
$$P_{jv}^{KE} = \left[\alpha_{jv}^E \left(P_{jv}^{EP} \right)^{1-\sigma_{jv}^{KE}} + \alpha_{jv}^K \left(\frac{R_{jv}}{\lambda_{jv}^K} \right)^{1-\sigma_{jv}^{KE}} \right]^{\frac{1}{1-\sigma_{jv}^{KE}}}$$

(2-15.3)
$$P_{jv}^{KEL} = \left[\alpha_{jv}^L \left(W_j^A \right)^{1-\sigma_{jv}^{KEL}} + \alpha_{jv}^{KE} \left(P_{jv}^{KE} \right)^{1-\sigma_{jv}^{KEL}} \right]^{\frac{1}{1-\sigma_{jv}^{KEL}}}$$

(2-15.4)
$$PXv_{jv} = \left[\alpha_{jv}^N \left(P_j^N \right)^{1-\sigma_{jv}^P} + \alpha_{jv}^{KEL} \left(P_{jv}^{KEL} \right)^{1-\sigma_{jv}^P} \right]^{\frac{1}{1-\sigma_{jv}^P}}$$

(2-15.5)
$$PX_j XP_j = \sum_v PXv_{jv} XPv_{jv}$$

(2-15.6)
$$PP_i XP_i = PX_i (1 + \tau_i^P) XP_i$$

3.5   Equilibrium in the Factor Markets

This section describes the determination of factor market equilibria. There are two parts to this section: the labour markets, and the capital markets.

There are as many labour markets as there are labour skills. Labour demand by skill type is determined by production decisions. A simple labour supply curve is implemented in Equation (2-16.1), with labour supply a function of the real wage.

Table 2-16: **Equilibrium on the Labour Markets**

$$(2\text{-}16.1) \qquad L_l^S = a_l \left(\frac{W_l}{P^{index}} \right)^{\omega_l}$$

$$(2\text{-}16.2) \qquad L_l^s = \sum_i L_{i,l}^d$$

Equation (2-16.2) is the market clearing condition and determines the equilibrium on the labour markets.

For the capital market it is necessary to distinguish between comparative statics and recursive dynamics.

Table 2-17: **Equilibrium on the Capital Market (comparative static)**

$$(2\text{-}17.1) \qquad \begin{cases} R^A = \left[\sum_i \alpha_i^k \left(R_i^{Old} \right)^{1+\omega^k} \right]^{1/(1+\omega^k)} & \text{if } \omega^k < \infty \\[3mm] K^s = \sum_i K_i^d & \text{if } \omega^k = \infty \end{cases}$$

$$(2\text{-}17.2) \qquad \begin{cases} K_i^{s,Old} = \alpha_i^k \left(\frac{R_i^{Old}}{R^A} \right)^{\omega^k} K^s & \text{if } \omega^k < \infty \\[3mm] R_i^{Old} = R^A & \text{if } \omega^k = \infty \end{cases}$$

$$(2\text{-}17.3) \qquad K_i^{s,Old} = K_i^d$$

In comparative static mode, all the dynamic transition equations are left out of the model definition. The putty/semi-putty structure of production is also irrelevant, and only *old* capital exists (i.e. only the *old* production elasticities are used). The sectoral supply of capital is determined using a CET supply function. An elasticity of substitution of zero implies sector-specific capital, and an elasticity of infinity implies perfectly mobile capital[19].

The equations in Table 2-17 determine sectoral capital supply in comparative static mode. In the case of finite elasticities, Equation (2-17.1) determines the aggregate (or average) rental rate using the definition of the CET dual price function. Equation (2-17.2) determines the sector-specific capital supply as a function of the sector specific rental rate

---

[19] In the current comparative static version of the model capital is assumed to be perfectly mobile across sectors.

relative to the average rate of return. Equation (2-17.3) determines the sector-specific rental rate through a market equilibrium equation. If the CET elasticity is zero, it is easy to see through Equation (2-17.2) that capital supply then becomes sector-specific.

If capital is perfectly mobile, i.e. the CET elasticity is infinite, Equation (2-17.1) determines the economy-wide (i.e. uniform) rate of return on capital, in other words, this equation is a market equilibrium equation. Equation (2-17.2) trivially sets the sectoral rental rate to the uniform rate, and Equation (2-17.3) trivially equates sector supply to sector demand.

In a long-term model, profit rates across sectors should be equal and therefore capital is usually assumed to be perfectly sectorally mobile. In a short-term model, the opposite is observed, i.e. sectors register different rates of profitability, and this leads to model capital as sector specific. The recursive dynamic framework used in the model allows us to have an intermediate situation between these two extremes by combining short term capital immobility (or low degree of mobility) for the old (or installed) vintage of capital and long-term perfect mobility for new capital.

In order to do that it is first necessary to determine the supply of old capital. At the beginning of a period if a sector is expanding, its supply of old capital, KO^s, is insufficient to produce its expanding output and therefore it will demand new capital. In this case it is assumed that the rental price of the old capital is the same as the rental price of the new capital. There is a unique economy-wide rental rate on new capital. If, however, a sector is declining, it will want to disinvest its beginning of period capital stock. In the case of a declining sector, the rental rate on old capital is sector specific. The disinvestment function is based on the relative rates of return of old capital versus new capital. The following equation determines the supply of old capital to a sector in decline:

$$KOld_{i,t}^s = KO_{i,t}^s \left[\frac{r_{i,t}^{Old} / r_{i,t}^{New}}{r_{i,t-1}^{Old} / r_{i,t-1}^{New}} \right]^{\eta_i^k}$$

Supply of old capital[20] will increase with the rental rate of old capital, with an absolute limit when the rental ratio of old and new capital is equal to 1. The equilibrium condition for old capital is that supply must equal demand, therefore, in the equation above, we replace directly the supply for old capital by demand for old capital, in other words the above equation is combined with the equilibrium equation. Finally, the above equation is inverted and solved for the rental ratio. This leads to Equation (2-18.1) in Table 2-18, where R^R is the ratio of the rental rate of old capital to the rental rate of new capital, and η^k is the disinvestment elasticity. The rental rate ratio is bounded above by 1.

---

[20] It is possible by simply subtracting $KO_{i,t}^s$ form both sides of the above equation, to represent the supply of disinvested capital as: $KO_{i,t}^s - KOld_{i,t}^s = KO_{i,t}^s \left[1 - \left(\frac{r_{i,t}^{Old} / r_{i,t}^{New}}{r_{i,t-1}^{Old} / r_{i,t-1}^{New}} \right)^{\eta_i^k} \right]$.

17

Table 2-18: **Supply of Old Capital in Declining Sectors**

$$(2\text{-}18.1) \qquad R^R_{i,t} = \min\left(R^R_{i,t-1}\left[\frac{Kv^d_{i,old,t}}{KO^s_{i,t}}\right]^{1/\eta^t_i} ,1 \right)$$

The single rental rate on all capital, which is not part of a declining sector, remains to be calculated. In other words, the single rental rate which applies to all new capital, plus old capital in expanding sectors, plus old capital being disinvested by declining sectors has not yet been determined. Equation (2-19.1) determines the rental rate of new capital, R^A, which is a single economy-wide rental rate. Equation (2-19.2) determines the sector specific rental rate of old capital. This could be determined as well by an equilibrium condition, but this was already integrated into Equation (2-19.1). Therefore the rental rate of old capital is simply determined by multiplying the rental rate ratio, by the rental rate of new capital. Finally, Equation (2-19.3) sets the rental rate of new capital.

Table 2-19: **Equilibrium on the Capital Market (recursive dynamics)**

$$(2\text{-}19.1) \qquad \sum_i K^d_{i,t} + K^d_{g,t} = K^s_t$$

$$(2\text{-}19.2) \qquad R^{Old}_i = R^A\, R^R_i$$

$$(2\text{-}19.3) \qquad R^{New}_i = R^A$$

3.6 Determination of Vintage Output

In each period, producers are faced with the decision to optimally allocate production across vintages. The model implements a simple rule. First, producers will use all the capital installed at the beginning of the period, i.e. old capital. If demand for output is greater than what can be produced with the installed capital, producers will demand new capital to produce the residual amount. If demand is less than what producers desire to produce with the installed capital, i.e. if a sector is in decline, producers will market the surplus capital on the second-hand market. The production allocation decision of the producer can be derived from the optimal capital/output ratio for each type of capital. The optimal capital/output ratio can be derived from Equations (2-9.3), (2-10.3), and (2-11.2). Equation (2-20.1) provides the capital/output ratio for each vintage type. The optimal capital/output ratio will depend on all the prices in the nested CES structure and will only be constant if the entire production structure is a Leontief technology. Equation (2-20.2) determines output produced by old capital. It uses the capital/output ratio to determine the optimal production with installed capital. If the latter is less than production, than that quantity is assigned to "old" production. If the quantity is greater than total demand, than the quantity produced with old capital will be set equal to total demand, and the residual

18

capital will be disinvested. Finally, Equation (2-20.3) determines the quantity of output produced with new capital. It will simply be the difference between total production and the amount produced with old capital.

Table 2-20: **Allocation of Domestic Production across Vintages**

$$(2\text{-}20.1) \qquad \chi_i^v = \alpha_{jv}^{KEL} \left(\frac{PXv_{jv}}{P_{jv}^{KEL}} \right)^{\sigma_{jv}^P} \alpha_{jv}^{KE} \left(\frac{P_{jv}^{KEL}}{P_{jv}^{KE}} \right)^{\sigma_{jv}^{kei}} \frac{\alpha_{jv}^k}{\lambda_{jv}^k} \left(\frac{\lambda_{jv}^k P_{jv}^{KE}}{R_{jv}} \right)^{\sigma_{jv}^{kv}}$$

$$(2\text{-}20.2) \qquad \begin{cases} XPv_i^{Old} = \dfrac{KO_i^s}{\chi_i^{Old}} & \text{if } \dfrac{KO_i^s}{\chi_i^{Old}} \le XP_i \\[3mm] XPv_i^{Old} = XP_i & \text{if } \dfrac{KO_i^s}{\chi_i^{Old}} > XP_i \end{cases}$$

$$(2\text{-}20.3) \qquad XPv_i^{New} = XP_i - XPv_i^{Old}$$

3.7 Income Distribution

Production generates income, both wage and non-wage, which is distributed to three main institutions: households, government and financial institutions (both domestic and foreign). In Table 2-21, Equation (2-21.1) determines operating surplus (net of depreciation allowance), Y^K. It is the sum across sectors and vintages of capital remuneration, and it incorporates factor payments from abroad. Equation (2-21.2) defines the depreciation allowance on the total capital stock of the previous period. Y^{DEPR} is the value of the depreciation allowance, δ is the depreciation rate (defined on the aggregate capital stock), and R^A is the economy-wide rental rate. Equation (2-21.3) defines company income, Y^{CORP}, it is equal to a share of net operating surplus (the rest being distributed to households and to foreigners). Equation (2-21.4) determines corporate direct taxes, C^{TAX}, with the base tax rate given by the parameter κ^C. Equation (2-21.5) defines retained earnings, i.e. corporate saving. Corporate saving is equal to a residual share of after-tax company income, net of transfer to the rest of the world.

Table 2-21: **Corporate Earnings Equations**

$$(2\text{-}21.1) \qquad Y^K = \sum_v \sum_t R_{tv} Kv_{tc}^d + E^{RATE} \sum_r F_r^K - Y^{DEPR}$$

$$(2\text{-}21.2) \qquad Y_t^{DEPR} = \delta \, R_t^A K_t$$

$$(2\text{-}21.3) \qquad Y^{CORP} = \chi^K Y^K$$

$$(2\text{-}21.4) \qquad C^{TAX} = \kappa^{CORP} Y^{CORP}$$

$$(2\text{-}21.5) \qquad S^{CORP} = \left(1 - \sum_h \phi_h^C \right) (1 - \kappa^{CORP}) Y^{CORP} - E^{RATE} \sum_r F_r^C$$

Household income derives from two main sources, capital and labour income. Additionally, households receive transfers from the government and from abroad. Equation (2-22.1) defines total labour income, Y^L as the product of total labour demand and the wage rate. There are two adjustments. One comes from wages earned abroad, the other concerns wages remitted to foreign labour. In the former case foreign wage income is assumed to be constant (in value), while, in the latter case, a fixed share (χ_l^F) of total domestic labour income is assumed to be distributed to foreign labour.

Labour income is distributed to the households. Equation (2-22.2) defines total households income, Y^H. It is the sum of labour income, distributed capital income and net company income, and transfers from the government T_h^G and from abroad F_{rh}^T. Capital and company income are distributed using fixed shares. Households direct tax, H^{TAX}, is given in Equation (2-22.3), where κ^H is the tax rate. The adjustment factor δ^{HTAX} becomes endogenous with the fixed government budget closure. In this case, when government revenues are changed during a policy simulation, the household tax schedule shifts in or out to achieve net government balance. Finally Equation (2-22.4) defines household disposable income Y^D. Disposable income is equal to total household income less taxes and transfer payments to the rest of the world.

Table 2-22: **Household Income**

(2-22.1)
$$Y_l^L = \chi_l^F \sum_i \omega_{lj} W_l L_{lj}^d + E^{RATE} \sum_r F_{rl}^L$$

(2-22.2)
$$Y_h^H = \sum_l \Xi_{h,l} Y_l^L + \phi_h^K \chi^h Y^K + \phi_h^C \left(1 - \kappa^{CORP}\right) Y^{CORP}$$
$$+ P^{Index} T_h^G + E^{RATE} \sum_r F_{r,h}^{Tin}$$

(2-22.3)
$$H_h^{TAX} = \delta^{HTAX} \kappa_h^H Y_h^H$$

(2-22.4)
$$Y_h^D = Y_h^H - H_h^{TAX} - E^{RATE} \sum_r F_{r,h}^{Tout}$$

3.8 Trade Equations

Import Structure

Demand by all economic agents has now been specified at the Armington level of aggregation, i.e. a composite demand of goods produced domestically and imports. In the current model, it is assumed that import demand by all agents is identical, in other words, we will not differentiate between producer's and consumer's propensity to import. This implies that we can aggregate the Armington demand across all agents before

determining the optimal allocation of the Armington demand between domestic goods and imported goods[21].

Recall that, the Armington assumption simply posits that goods are differentiated with respect to region of origin. The model has implemented this assumption using a nested structure. At the top level, each domestic agent optimises some objective function (e.g. cost minimisation or utility maximisation). This leads to demand for a composite commodity that has been referred to as the Armington commodity. At the next level, agents minimise the cost of the Armington bundle, subject to an aggregation function between goods produced domestically and an aggregate import bundle. In the case of this model, this is a CES aggregation function. At the next and final level, agents minimise the cost of the aggregate import bundle, again subject to an aggregation function over imports originating in each region of the model, namely the EC region the other European countries region and the Rest of the World.

The mathematical formulation leads to:

$$\min P^D D + P^M M$$

s.t.

$$X = \left[a_d D^\rho + a_m M^\rho \right]^{1/\rho}$$

where X is the demand for the Armington good, D is demand for domestic production, M is aggregate import demand, P^D is the price of domestic sales, and P^M is the domestic price of imports (tariff inclusive).

The first order conditions lead to the following demand functions:

$$D = \alpha_d X \left(\frac{PA}{P^D} \right)^\sigma \quad \text{where} \quad \alpha_d = a_d^\sigma$$

$$M = \alpha_m X \left(\frac{PA}{P^M} \right)^\sigma \quad \text{where} \quad \alpha_m = a_m^\sigma$$

and the substitution elasticity is given by:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma - 1}{\sigma}$$

PA is the (Armington) CES dual price determined using PD and PM:

$$PA = \left[\alpha_d P^{D^{(1-\sigma)}} + \alpha_m P^{M^{(1-\sigma)}} \right]^{\frac{1}{1-\sigma}}$$

---

[21] This is done in according to what shown in chapter on the multipliers. In addition, this formulation is less data intensive.

Table 2-23 specifies the equations relative to the first level of the Armington nest. Equation (2-23.1) determines aggregate Armington demand, XA, i.e. the sum of Armington demand across all agents. Equations (2-23.2) and (2-23.3) decompose the aggregate Armington demand into respectively the domestic component, XD, and the aggregate import component, XM.

Table 2-23: Armington Decomposition

$$(2\text{-}23.1) \qquad XA_i = \sum_j X_{i,j}^{AP} + \sum_h X_{ih}^{AC} + \sum_f X_{if}^{AF}$$

$$(2\text{-}23.2) \qquad X_i^D = \alpha_i^d \, XA_i \left(\frac{PA_i}{P_i^D} \right)^{\sigma_i^d}$$

$$(2\text{-}23.3) \qquad X_i^M = \alpha_i^m \, XA_i \left(\frac{PA_i}{P_i^M} \right)^{\sigma_i^m}$$

Equation (2-24.1) in Table 2-24 determines the Armington price, PA, which is the CES dual price of the Armington component prices, i.e. P^D and P^M.

Table 2-24: Armington Price

$$(2\text{-}24.1) \qquad PA_i = \left[\alpha_i^d \, P_i^{D(1-\sigma_i^m)} + \alpha_i^m \, P_i^{M(1-\sigma_i^m)} \right]^{\frac{1}{1-\sigma_i^m}}$$

The next stage in the Armington decomposition is to decompose the aggregate Armington demand across the regions of the model. Again, the CES functional form is used to implement the imperfect substitutability of commodity demand across regions.

Equation (2-25.1) in Table 2-25 determines import volume by sector and region of origin, $X_{r,i}^{MReg}$. The relevant import price, is the partner-specific import price $P_{r,i}^{MReg}$, in domestic currency and inclusive of partner-specific import tariff. Equation (2-25.2) determines the price of the aggregate import bundle, P_i^M, which is the CES dual price. Equation (2-25.3) defines the domestic import price, $P_{r,i}^{MReg}$, which is equal to the import price of the trading partner converted in local currency, and inclusive of the partner-specific tariff rate.

22

Table 2-25: **Second-Level Armington Equations**

$$(2\text{-}25.1) \qquad X_{r,i}^{MReg} = \beta_{r,i}^{Reg} \left(\frac{P_i^M}{P_{r,i}^{MReg}} \right)^{\Lambda_i^r} X_i^M$$

$$(2\text{-}25.2) \qquad P_i^M = \left[\sum_r \beta_{r,i}^{Reg} \left(P_{r,i}^{MReg} \right)^{(1-\Lambda_i^r)} \right]^{\frac{1}{1-\Lambda_i^r}}$$

$$(2\text{-}25.3) \qquad P_{r,i}^{MReg} = E^{RATE} P_{r,i}^{World} \left(1 + \tau_{r,i}^M \right)$$

Export Structure

Export supply is treated symmetrically to import demand. Producers are assumed to differentiate between the domestic market and the export market. Producers are modelled as maximising sales between the domestic and export markets subject to being on a production possibilities frontier. The model uses the Constant Elasticity of Transformation (CET) specification to implement the production possibilities frontier. The resulting equations are similar to the CES first order condition with reversals in signs to reflect that producers are maximising revenues, as opposed to the CES where agents are minimising costs. As for the Armington specification, there are two levels in the export supply structure. Exporters are assumed to differentiate across regions, in response to changes in relative regional export prices.

Equation (2-26.1) and (2-26.2) provide the first order conditions for determining the producers supply decisions. Equation (2-26.1) determines the optimal supply of goods for the domestic market, X^D. Notice the change from the CES functional form. A rise in the domestic price (with respect to the producer price), leads to a rise in domestic supply. The computer implementation of the model allows for the possibility of infinite CET elasticities in which case the producer does not differentiate between domestic and foreign markets. Under this assumption, the domestic and foreign sales price are identically equal to the producer price. Equation (2-26.2) determines export supply, E^S. Equation (2-26.3) is the CET dual price function which replaces the primal CET function in the case of finite elasticities.[22] In the case of infinite elasticities, output is equal to the simple sum of domestic supply and export supply.

---

[22] The primal function is given by the following formula:

$$XP_i = \left[a_{t,i}^d X_i^{D^{\rho_i^t}} + a_{t,i}^e E_i^{S^{\rho_i^t}} \right]^{1/\rho_i^t}$$

where the following relations hold:

$$\rho_i^t = \frac{\sigma_i^t + 1}{\sigma_i^t} \Leftrightarrow \sigma_i^t = \frac{1}{\rho_i^t - 1} \qquad \text{and} \qquad a_{t,i}^d = \left(\alpha_{t,i}^d \right)^{1-\rho_i^t} \quad a_{t,i}^e = \left(\alpha_{t,i}^e \right)^{1-\rho_i^t}$$

Table 2-26: CET Decomposition

$$(2\text{-}26.1) \quad \begin{cases} X_i^D = XP_i \left(\dfrac{P_i^D}{\alpha_{t,i}^d PP_i} \right)^{\sigma_i^T} & \text{if} \quad \sigma_i^T < \infty \\[3mm] P_i^D = PP_i & \text{if} \quad \sigma_i^T = \infty \end{cases}$$

$$(2\text{-}26.2) \quad \begin{cases} E_i^S = XP_i \left(\dfrac{P_i^E}{\alpha_{t,i}^E PP_i} \right)^{\sigma_i^T} & \text{if} \quad \sigma_i^T < \infty \\[3mm] P_i^E = PP_i & \text{if} \quad \sigma_i^T = \infty \end{cases}$$

$$(2\text{-}26.3) \quad \begin{cases} PP_i = \left[\alpha_{t,i}^D P_i^{D\,\sigma_i^T+1} + \alpha_{t,i}^E P_i^{E\,\sigma_i^T+1} \right]^{1/\sigma_i^T+1} & \text{if} \quad \sigma_i^T < \infty \\[3mm] XP_i = X_i^D + E_i^S & \text{if} \quad \sigma_i^T = \infty \end{cases}$$

The final trade equations determine the second nest of the CET decomposition of exports. The regional disaggregation of exports is given in Equation (2-27.1). Equation (2-27.2) determines aggregate export price P^E.

Table 2-27: Second-Level CET Equations

$$(2\text{-}27.1) \quad E_{r,i}^{SReg} = \alpha_{r,i}^{Reg} E_i^S \left(\frac{PE_{r,i}^{Reg}}{P_i^E} \right)^{\Omega_i^r}$$

$$(2\text{-}27.2) \quad P_i^E = \left[\sum_r \alpha_{r,i}^{Reg} \left[PE_i^{Reg} \right]^{(1+\Omega_i^r)} \right]^{\frac{1}{1+\Omega_i^r}}$$

Table 2-28 presents the equations which determine export demand by the regional trading partners, and the export market equilibrium condition. Equation (2-28.1) defines export demand by the trading partner E^D. Since it is assumed that Colombia has no market power in its export markets (small country assumption), the export demand elasticity is infinite, i.e. it is a flat demand curve which determines the fixed price of exports. Equation (2-28.2) defines the export market equilibrium, i.e. the equality between export supply and foreign demand.

24

Table 2-28: **Export Demand and Market Equilibrium**

$$(2\text{-}28.1) \qquad PE_{r,i}^{Reg} = E^{RATE} \overline{P}_{r,i}^{EWorld}$$

$$(2\text{-}28.2) \qquad E_{r,i}^{SReg} = E_{r,i}^{DReg}$$

3.9 *Government Revenues and Saving, and Macro Closure*

Table 2-29 presents the government revenues and closure rules. Equations (2-29.1) and (2-29.2) determine respectively the government's tax revenues from the production tax and the import tax. Note that in Equation (2-29.2) the regional indices are explicitly used. The tariff rates are trading-partner specific, i.e. the tariff rate is allowed to vary depending on the region of import. Equation (2-29.3) identifies miscellaneous government revenues, these are all revenues minus household direct taxes. Equation (2-29.4) provides total nominal government revenues, G^{rev}.

Table 2-29: **Government Revenues and Saving**

$$(2\text{-}29.1) \qquad Y^{INDTAX} = \sum_i \tau_i^P PX_i XP_i$$

$$(2\text{-}29.2) \qquad Y^{TARIF} = E^{RATE} \sum_r \sum_i \tau_{r,i}^M P_{r,i}^{World} X_{r,i}^{MReg}$$

$$(2\text{-}29.3) \qquad Y^{MiscRev} = Y^{INDTAX} + Y^{TARIF} + C^{TAX} + E^{RATE} \sum_r F_r^{Gin}$$

$$(2\text{-}29.4) \qquad G^{Rev} = Y^{MiscRev} + \sum_h H_h^{TAX}$$

$$(2\text{-}29.5) \qquad S_g = P^{GDP} \overline{S}_g$$

$$(2\text{-}29.6) \qquad S_g = G^{Rev} - G^{Exp}$$

The government closure rule is specified in Equation (2-29.3): government saving is fixed (in real terms). Government saving is simply the difference between government revenue and government expenditure and is determined by Equation (2-29.4). With exogenous government saving the household tax schedule is endogenous. Household taxes are determined by solving Equation (2-22.3) for δ^{HTAX}, i.e δ^{HTAX} is the equilibrating variable to achieve the fixed government balance.[23]

Table 2-30 includes the equations for the closure of the saving and investment account, and other macro closures. Domestic investment is equal to domestic saving plus a fixed level of foreign saving. Under this rule, foreign saving does not react to regional

---

[23] In the reference scenario, government saving is held fixed at its base year level which implies that, as a share of GDP, government saving (or the deficit) declines over time.

changes in relative rates of return. In this case, the *value* of investment, D_{inv}^{VTFD}, is determined by Equation (2-30.1). Aggregate investment (in value) is the sum of saving, plus depreciation. Saving includes retained corporate earnings, S^{CORP}, household saving, S^{H}_{Tot}, government saving, S_g, and total foreign saving, $\sum_r S_r^F$. Foreign savings are exogenous in each time period.

Table 2-30: **Determination of Aggregate Investment and Macro Closure**

$$(2\text{-}30.1) \qquad D_{inv}^{VTFD} = S^{CORP} + S_{Tot}^{H} + S_g + E^{RATE}\sum_r S_r^F + Y^{DEPR}$$

$$(2\text{-}30.2) \qquad E^{RATE}\sum_i\sum_r P_{r,i}^{World} X_{r,i}^{MReg} = \sum_i\sum_r PE_{r,i}^{Reg} E_{r,i}^{SReg} + E^{RATE}\sum_r S_r^F$$

$$-\sum_l\left(1-\chi_l^F\right)\sum_i \omega_{lj} W_l L_{lj}^d + E^{RATE}\sum_r F_{rl}^L$$

$$-\chi^r Y^K + E^{RATE}\sum_r F_r^K - E^{RATE}\sum_r F_r^C$$

$$-E^{RATE}\sum_r\sum_h\left(F_{r,h}^{Tout}-F_{r,h}^{Tin}\right) - E^{RATE}\sum_r\left(F_r^{Gout}-F_r^{Gin}\right)$$

$$(2\text{-}30.3) \qquad X^{GDP} = \sum_l W_l \sum_l \omega_{li} L_{li}^d + \sum_v\sum_i R_{iv} K v_{iv}^d$$

$$(2\text{-}30.4) \qquad X^{RGDP} = \sum_l W_{l,0}\sum_l \omega_{li}\lambda_{li}^L L_{li}^d + \sum_v\sum_i R_{iv,0}\lambda_{iv}^K K v_{iv}^d$$

$$(2\text{-}30.5) \qquad P^{Index} = \frac{X^{GDP}}{X^{RGDP}}$$

In the model, Walras law has been defined to be the equality of the trade balance and the (negative) of foreign saving. Equation (2-30.2) defines Walras law. On one side of the balance sheet are exports, evaluated at world prices, net total foreign saving, and net transfers and factor payments. On the other side of the balance sheet is the sum of imports evaluated at world prices (excluding tariffs). Due to Walras' Law, one equation is redundant, and Equation (2-30.2) is dropped from the model.

The final equations of the model, Equations (2-30.3)-(2-30.5) are used to calculate the domestic price index which is used to inflate real domestic transfers. Equation (2-30.3) calculates the current value GDP, X^{GDP}. Equation (2-30.4) defines real GDP, X^{RGDP}, as the sum of factor demands in efficiency units, evaluated at base year prices. Equation (2-30.5) defines the GDP deflator, P^{Index}. The GDP deflator is defined as the value of factor payments, divided by the sum of factor volumes.

Any price in the model can be chosen as the numéraire. In the current version of the model, the foreign price index, E^{RATE}, has been designated as the numéraire, and its value is always set to 1.

3.10 Aggregate Capital Stock and Productivity Growth

This section, and the next, provide the key equations for describing the transition from one period to the next. The aggregate capital stock is not pre-determined because it depends on the current level of investment. The one-year gap transition equation is given by:

$$K_t = (1-\delta)K_{t-1} + I_{t-1}$$

where K is the aggregate capital stock, δ is the annual rate of depreciation, and I_{t-1} is the level of real investment in the previous period. A problem appears when the gap between solution periods is greater than 1 year. Since investment in the intervening years is not calculated assumptions must be made in order to integrate the stream of investment. The transition equation for a multi-period gap expanded has this form:

$$K_t = (1-\delta)\left[(1-\delta)K_{t-2} + I_{t-2}\right] + I_{t-1}$$
$$\vdots$$
$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n}(1-\delta)^{j-1} I_{t-j}$$

The model does not calculate investment between periods. A linear growth model is assumed to explain investment in intermediate years, i.e.:

$$I_j = (1+\gamma^i)I_{j-1}$$

where

$$\gamma^i = \left(\frac{I_t}{I_{t-n}}\right)^{\frac{1}{n}} - 1$$

where the annual growth rate of investment is derived from the annualised growth rate of investment in the current period compared to investment in the previous period. We can re-write the multi-year transition equation to be:

$$K_t = (1-\delta)^n K_{t-n} + \sum_{j=1}^{n}(1-\delta)^{j-1}(1+\gamma^i)^{n-j} I_{t-n}$$

The transition equation is then derived and given by Equation (2-31.2) in Table 2-31, where the growth parameter γ^i is determined in Equation (2-31.1). The capital stock is only pre-determined in Equation (2-31.2) if the gap between periods is equal to one year. Due to base year normalisation rules (the rental rate is set to 1 in the base year), the aggregate stock of capital, K, is normalised to yield K^s (Equation (2-31.3)), which is the level of capital used in determining equilibrium on the capital market.[24]

---

[24] The following numerical example may shed some light on the normalisation rule. Assume the value of capital in a region is 100. Assume, as well, that capital remuneration is 10. Capital remuneration is simply rK where r is the rental rate and K the demand for capital. In this example, rK is equal to 10, which implies a rental rate of of 0.1. EMMA uses a different normalisation rule. It assumes that the base year rental rate is 1, and normalises the capital data to be consistent with this normalisation rule, in other words, the

Table 2-31: **Aggregate Capital Stock**

$$(2\text{-}31.1) \qquad \gamma^i = \left(\frac{I_t}{I_{t-n}} \right)^{\frac{1}{n}} - 1$$

$$(2\text{-}31.2) \qquad K_t = (1-\delta)^n K_{t-n} + \frac{(1+\gamma^i)^n - (1-\delta)^n}{\gamma^i + \delta} I_{t-n}$$

$$(2\text{-}31.3) \qquad K_t^s = \frac{K_{t-n}^s}{K_{t-n}} K_t$$

Productivity

The efficiency growth of labour and energy is always assumed to be exogenous. The efficiency growth of capital is normally exogenous, but in the reference scenario, the capital efficiency factor is calibrated in order to achieve a target growth rate for real GDP. Since there is only one target growth rate for real GDP per region, there can only be one instrument to achieve this target. In the current version of the model, it is assumed that capital efficiency is uniform across sectors and vintages. The capital efficiency parameter is only endogenous in the reference (or business-as-usual) scenario. In all shock simulations, the capital efficiency parameter is exogenous.

Equation (2-32.1) determines the real growth rate of GDP, γ^y, in most simulations. However, in the reference simulation, γ^y isexogenous, and Equation (2-32.1) is used to determine the capital efficiency growth parameter, γ^k. Equation (2-32.2) determines the cumulative capital efficiency factor.

Table 2-32: **Productivity Factors for Capital**

$$(2\text{-}32.1) \qquad X_t^{RGDP} = \left(1 + \gamma^y\right)^n X_{t-1}^{RGDP}$$

$$(2\text{-}32.2) \qquad \lambda_{jv,t}^k = \left(1 + \gamma_t^k\right)^n \lambda_{jv,t-n}^k$$

The remaining equations deal with the pre-determined variables, which are updated at the beginning of each period. These are transition equations do not rely on any contemporaneous variable, hence are not directly an endogenous result of the model.

---

normalised capital demand is 10, and it is really an index of capital volume. The non-normalised level of capital is used only in the accumulation function (Equation 16.1.2), and in determining the value of capital depreciation allowance. All other capital stock equations use the normalised value of capital.

<div align="center">

Table 2-33: Initial Supply of Old Capital

</div>

(2-33.1) $KO_{i,t}^{s} = (1 - \delta)^{n} K_{i,t-n}^{d}$

In Table 2-33 KO^{s} represents the installed old capital at the beginning of each period by sector.[25] It is simply equal to the sector's previous period's total (depreciated) capital stock. The end of period stock of old capital (in a given sector) may be less than the initial stock. If the sector is declining, old capital will be disinvested and the actual stock of old capital will be less than the initial stock.

<div align="center">

Table 2-34: Other Pre-Determined Exogenous Variables

</div>

(2-34.1) $a_{l,t} = (1 + \gamma_{t}^{l})^{n} al_{,t-n}$

(2-34.2) $Pop_{t} = (1 + \gamma_{t}^{p})^{n} Pop_{t-n}$

(2-34.3) $D_{g,t}^{TFD} = (1 + \gamma_{t}^{y})^{n} D_{g,t-n}^{TFD}$

In Table 2-34, Pop_{t} is the population at time t. D_{g}^{TFD} is the level of total real government expenditures on goods and services is assumed to grow at the same rate as the economy.[26] Equation (2-34.1) determines the labour supply shift factor which is equal to the previous period's labour supply shift factor multiplied by an exogenously specified labour supply growth rate.

The energy efficiency factors are also exogenous and pre-determined leading to the following set of transition equations:

<div align="center">

Table 2-35: Energy and Labour Efficiency Factors

</div>

(2-35.1) $\lambda_{i,t}^{E} = (1 + \gamma_{i,t}^{E})^{n} \lambda_{i,t-n}^{E}$

(2-35.2) $\lambda_{l,t}^{l} = (1 + \gamma_{t}^{l})^{n} \lambda_{l,t-n}^{l}$

The annual autonomous energy efficiency factor is given by $\gamma^{e,p}$, representing the growth in energy efficiency in production. The energy efficiency factors in production

---

[25] The base year data does not provide capital stock data by sector only capital remuneration by sector. Since the price of capital in each sector is set to 1 in the base year, the sectoral capital stock data should be seen as an index volume and not a volume, which has been observed. Since it is further assumed that the aggregate capital stock is the sum of the sectoral capital stocks, this implies that the price of capital is the same across sectors in the base year.

[26] Note that the real level of transfers from government to households is assumed to grow at the same rate as the economy, too.

are specific to both sector and vintage. The cumulative factor is given by the λ variable. Equation (16.6.2) determines the labour efficiency factor. The growth in labour efficiency is exogenous and is labour-type specific.

Vintage Re-Calibration

The model has a vintage structure of capital which is based on an assumption of a putty/semi-putty structure of production. It is further assumed that the substitutability of capital differs across vintage, with old capital typically less substitutable than new capital. There are only two vintages, *old* and *new*. New capital is generated by investment in the previous period. Old capital is the installed capital in the previous period. Over time, the structure of old capital changes as the previously new capital gets merged into the old capital. Rather than keep track of each vintage over time, we modify the structural parameters of the old capital to reflect its changing composition. The key rule that has been adopted is that the share parameters associated with old capital should be able to produce all of the previous period's production (with the substitution elasticities of the old capital). For example, assume we have a CES production function in capital (K), labour (L), and energy (E). Production then has the form:

$$X_v = [a_{k,v} K_v^{\rho_v} + a_{l,v} L_v^{\rho_v} + a_{e,v} E_v^{\rho_v}]^{1/\rho_v}$$

where

$$\sigma_v = \frac{1}{1-\rho_v}$$

and X is output (by vintage), K_v is capital by vintage, L_v is labour, and E_v is energy. The share parameters are vintage specific as is the substitution elasticity. The first order conditions for cost minimisation lead to:

$$K_v = \alpha_{k,v} X_v \left(\frac{P_v}{r_v}\right)^{\sigma_v} \quad \text{where} \quad \alpha_{k,v} = a_{k,v}^{\sigma_v}$$

$$L_v = \alpha_{l,v} X_v \left(\frac{P_v}{w}\right)^{\sigma_v} \quad \text{where} \quad \alpha_{l,v} = a_{l,v}^{\sigma_v}$$

$$E_v = \alpha_{e,v} X_v \left(\frac{P_v}{e_v}\right)^{\sigma_v} \quad \text{where} \quad \alpha_{e,v} = a_{e,v}^{\sigma_v}$$

where r, w, and e are respectively the price of capital, labour, and energy. (Note that the model uses the modified share parameters, i.e. the share parameters of the CES function raised to the power of the substitution elasticity. The former share parameters are never formally employed in the model since only the first order conditions and the CES price function are used.) P is the CES dual price which is given by the following equation:

$$P_v = \left[\alpha_{k,v} r_v^{1-\sigma_v} + \alpha_{l,v} w^{1-\sigma_v} + \alpha_{e,v} e_v^{1-\sigma_v}\right]^{1/(1-\sigma_v)}$$

We assume that the production structure of output associated with new capital is constant over time, i.e. the share parameters and substitution elasticities are not time

30

dependent (except for the efficiency factors). However, old capital changes over time as in each time period previously new capital is added to the old capital stock. In order to account for the change in old capital the share parameters for the production structure associated with old capital are modified in such a way that the total of the factors in the previous period could produce all of the previous period's output assuming the old substitution elasticities. To continue with the above notation, we re-calibrate the share parameters according to the following formula:

$$\overline{\alpha}_{k,o} = \frac{K_{t-1}}{X_{t-1}} \left(\frac{r_{t-1}}{P_{t-1}} \right)^{\sigma_o}$$

$$\overline{\alpha}_{l,o} = \frac{L_{t-1}}{X_{t-1}} \left(\frac{w_{t-1}}{P_{t-1}} \right)^{\sigma_o}$$

$$\overline{\alpha}_{e,o} = \frac{E_{t-1}}{X_{t-1}} \left(\frac{e_{t-1}}{P_{t-1}} \right)^{\sigma_o}$$

where the re-calibrated share parameters, α , are calibrated at the beginning of each period, and all the volumes are the sum of the old and new vintages from the previous period, and the prices are the average prices (N.B. the subscript o is used for old capital, and n for new capital):

$$X_{t-1} = X_{o,t-1} + X_{n,t-1}$$

$$K_{t-1} = K_{o,t-1} + K_{n,t-1}$$

$$E_{t-1} = E_{o,t-1} + E_{n,t-1}$$

$$P_{t-1} = \left[P_{o,t-1}X_{o,t-1} + P_{n,t-1}X_{n,t-1} \right] / X_{t-1}$$

$$r_{t-1} = \left[r_{o,t-1}K_{o,t-1} + r_{n,t-1}K_{n,t-1} \right] / K_{t-1}$$

$$e_{t-1} = \left[e_{o,t-1}E_{o,t-1} + e_{n,t-1}E_{n,t-1} \right] / E_{t-1}$$

With the above definitions of the aggregate factors and average factor prices, the production function associated with the α parameters is consistent with the aggregate output X_{t-1}.

For brevity, the above formulas are not repeated for all the nested CES production functions. As described in more detail below, the production structure can be represented by a nested tree structure of CES and Leontief functions. Within this structure, there are several CES aggregation functions whose old-vintage share parameters are re-calibrated in the manner described above.

4   An illustrative example of policy evaluation with COGEM

In this section we show a simple application of the model to evaluate the effects of a reduction in government expenditures. This should be considered an illustrative example and not necessarily as an exhaustive investigation in Colombian fiscal adjustment policy nor a policy recommendation of the authors.

Due to the recent international agencies' evaluations on the effects of structural adjustment reforms[27] and internal economic policy debate, there has been in Colombia an intense renewed discussion concerning the structural policies to be implemented in the next 5 to 10 years. Among these, a prominent reform deals with an adjustment in public finance.

Given that a major factor affecting the government budget has been identified with increased public expenditure[28], we simulate with the model a policy of reduction in public consumption. Figures

Figure 6-1 shows the exact amount of this reduction in terms of millions of current pesos of public consumption and as a ratio of public consumption on GDP. The proposed reduction is equivalent to a 2.5 % yearly real contraction in public expenditure, which should not be thought of as disproportionate given that recent increases in this same variable have been three times larger. With this policy, the Colombian government (excluding public enterprises) will reach a balanced budget in 8 years.

The aggregate results of this policy are shown in figures 7.2, 7.3 and 7.4. The main striking results is that GDP growth rate will be, by the final year of our simulation, 0.3 points above the level projected in the scenario with no reforms. Two major mechanisms operating in the model explain this increased growth rate. Firstly, and most importantly, during the current simulation public expenditure is substituted by private expenditure in a sort of crowding-in effect. In fact decreased public spending allows to reduce budget deficit and, at the same time, fiscal pressure on private agents. These take advantage of the relaxed budget constraint by increasing their consumption and savings. In this way the whole economy's aggregate savings increase in this simulation. This allows a larger volume of investments and a greater capital accumulation.

The second mechanism is a direct consequence of the increased *new vintage* capital stock. We saw in the previous section that *new vintage* capital stock has a higher degree of substitutability with the other production factors, therefore the larger this type of capital, the more flexible the production structure. Increased flexibility results in a better allocation of resources or, in other words, in a more efficient/productive economy.

It should be noticed that these two mechanisms, increased savings and increased efficiency, explain the positive growth differential. We do not take into account other important pro-growth effects such as productivity differences between private and public expenditures and the probable increased external capital inflows attracted by sound budget policy.

The described policy positive effects on private investment and consumption are depicted in Figure 6-3, which shows percentage differentials in the values for these two variables measured in the business-as-usual scenario and in the fiscal adjustment policy scenario. In this latter case, we can see that initially (for 1998) investment slightly lags behind consumption, to reach, by the final year, a value of 10 per cent larger than in the business-as-usual case.

---

[27] See IADB 1997 report, or World Bank 1997 World Development report, or S.J. Burky and G. Perry (1997) book.
[28] See Fedesarrollo (1998), page 58-69.

The final aggregate results shown here concern the labour market. If implemented the examined policy may increase labour demand growth rate by almost 0.2 points, corresponding to more than 250,000 new jobs, as shown in Figure 6-4.

Clearly many other more detailed results, such as sectoral outputs, trade flows, income distribution changes, price indices, are available but for brevity are not commented here.

5 Conclusion

This paper described the algebraic structure of the model used in the study of the Colombian economic policy analyses. Although some of its characteristics are quite conventional in the literature on these type of models some features stands out as particularly innovative.

Firstly its level of details: this model accommodates 23 sectors, 10 household types, 3 labour categories, 3 disaggregated trade partners. A second peculiar characteristic is the use of putty-semi putty structure of production. This lets us model with separate elasticity of substitution two capital vintages representing old and new investment goods, and, in turn, adds realism to applied work. Finally this model by including differentiated household groups allows one to study the effects policy changes on income distribution.

6 Bibliography

What follows is an informational bibliography of references related to applied general equilibrium modeling. All these materials bear upon the methods which will be used to construct the Colombian CGE facility, and they illustrate the scope of methods and policy applications for these models.

Adelman, I., and S. Robinson (1978), **Income Distribution Policies in Developing Countries**, Stanford University Press, Stanford.

Adelman, I., D.W. Roland-Holst, and A. Sarris (1989), "Adjustment Under Uncertainty with Calibrated General Equilibrium Models," **International Economic Journal**.

Ballard, C., D. Fullerton, J. Shoven and J. Whalley (1985), **A General Equilibrium Model for Tax Policy Evaluation**, Chicago: University of Chicago Press for NBER.

Benjamin, N., and S. Devarajan (1985), "Oil Revenues and Economic Policy in Cameroon: Results fro a Calibrated General Equilibrium Model," World Bank Staff Working Paper No. 745, World Bank, Washington, D.C.

Bergman, L., and L. Ohlsson (1981), "Changes in Comparative Advantage and Paths of Structural Adjustment and Growth in Sweden, 1975-2000," Research Report RR-81-13, International Institute of Applied Systems Analysis, Laxenburg, Austria.

Borges, A.M. (1986), "Applied General Equilibrium Models: An Assessment of their Usefulness for Policy Analysis," **OECD Economic Studies**, 7, pp.8-43.

Bourguignon, F., W.H. Branson, and J. de Melo (1989), "Adjustment and Income Distribution: A Counterfactual Analysis," Working Paper, The World Bank, Washington, D.C.

Bovenberg, A.L. (1989), "The Effects of Capital Income Taxation on International Competitiveness and Trade Flows," **American Economic Review**, 79, forthcoming.

Brooke, A., D. Kendrick, and A. Meeraus (1988), **GAMS: A User's Guide**, Palo Alto: The Scientific Press. (Redwood City, CA)

Brown, D.K. (1988), "Tariffs, the Terms of Trade and National Product Differentiation," **Journal of Policy Modelling**.

Brown, D.K., and R.M. Stern (1988), "U.S.- Canada Bilateral Tariff Elimination: The Role of Product Differentiation and Market Structure," Discussion Paper No. 227, Department of Economics, University of Michigan, Ann Arbor.

Brown, F., and J. Whalley (1980), "General Equilibrium Evaluations of Tariff-Cutting Proposals in the Tokyo Round and Comparisons with More Extensive Liberalization of World Trade," **The Economic Journal**, 90:360, pp.838-866.

Bruniaux, J-M, F. Delorme, I. Lienert, J.P. Martin, and D. Roland-Holst, 1988, "Economywide Effects of Agricultural Policies in OECD Countries: Simulation Results with WALRAS," OECD Economic Studies, Fall.

Bussolo, M, R. Correa, S. Prada, 1998, "A Detailed Social Accounting Matrix for Colombia 1994", FEDESARROLLO working paper, forthcoming.

Condon, T., V. Corbo, and J. de Melo (1985), "Productivity Growth, External Shocks, and Capital Inflows in Chile During 1977-81: A General Equilibrium Analysis," **Journal of Policy Modeling**, 7:3, pp.379-405.

Cox, D. and R. Harris (1985), "Trade Liberalization and Industrial Organization: Some Estimates for Canada," Journal of Political Economy, Vol. 93, No. 1, pp. 115-145.

Dahl, H., S. Devarajan, and S. van Wijnbergen (1989), "Revenue Neutral Tariff Reform: Theory and An Application to Cameroon," CPD Discussion Paper No. 1986-26, World Bank, Washington, DC.

Deardorff, A. and R. Stern (1986), **The Michigan Model of World Production and Trade**, MIT Press, Cambridge, Mass.

Deardorff, A.V., and R.M. Stern (1979), **An Economic Analysis of the Effects of the Tokyo Round of Multilateral Trade Negotiations on the United States and the Other Major Industrialized Countries**, MTN Studies 5, U.S. Government Printing Office, Washington, D.C.

Dervis, Kemal, Jaime de Melo and Sherman Robinson (1982), **General Equilibrium Models for Development Policy,** Cambridge: Cambridge University Press.

Devarajan, S., and H. Sierra (1985), "Growth without Adjustment: Thailand, 1973-82," processed, Country Policy Department, World Bank, Washington, D.C.

Devarajan, S., C. Jones, and M. Roemer (1989), "Markets Under Price Controls in Partial and General Equilibrium," **World Development**, 12:12.

Devarajan, S., J.D. Lewis, and S. Robinson (1986), "A Bibliography of Calibrated General Equilibrium (CGE) Models Applied to Developing Countries," Gianinni Foundation Working Paper No. 400, University of California, Berkeley.

Dewatripont, M., and S. Robinson (1985), "The Impact of Price Rigidities: A Calibrated General Equilibrium Analysis," Gianinni Foundation Working Paper No. 375, University of California, Berkeley.

Dixit, A.K. (1985), "Tax Policy in Open Economies," in A. Auerbach and M. Feldstein, eds., **Handbook of Public Economics**, Amsterdam, North- Holland.

Dixon, P., B. Parmenter, J. Sutton and D. Vincent (1982), **ORANI: A Multi-Sector Model of the Australian Economy**, Amsterdam: North- Holland.

Ethier, W.J. (1971), "General Equilibrium Theory and the Concept of Effective Protection," in H.G. Grubel and H.G. Johnson (eds.), **Effective Tariff Protection**, Graduate Institute of International Studies, Geneva.

Evans, H.D. (1972), **A General Equilibrium Analysis of Protection**, North-Holland, Amsterdam.

Ezaki, M. (1985), "A Calibrated General Equilibrium Model of the Japanese Economy," Discussion Paper No. 120, Center for Southeast Asian Studies, Kyoto University, Kyoto.

FEDESARROLLO (1998). "Reformas Pendientes en la Economía Colombiana." In: **Prospectiva: Perspectivas Económicas de Corto y Mediano Plazo**, enero-junio, p. 58-69.

Feltenstein, A. (1980), "A General Equilibrium Approach to the Analysis of Trade Restrictions, with an Application to Argentina," **IMF Staff Papers**, 27:4, pp.749-784.

Fullerton, D. and Y.K. Henderson (1989), "A Disaggregate Equilibrium Model of the Tax Distortions Among Assets, Sectors, and Industries," **International Economic Review**, 30:2, pp.391-413.

Gelb, A., and P. Conway (1984), "Oil Windfalls in a Controlled Economy: A 'Fix-Price' Equilibrium Analysis of Algeria," Discussion Paper No. 94, Development Research Department, The World Bank, Washington, D.C.

Ghanem, H. (1985), "Senegal: A Study of Alternative Foreign Borrowing Strategies," Discussion Paper No. 1985-7, Country Policy Department, The World Bank, Washington, D.C.

Goulder, L. and B. Eichengreen (1989b), "Trade Liberalization in General Equilibrium: Intertemporal and Inter-industry Effects," NBER Working Paper 2695, Cambridge.

Goulder, L., and B. Eichengreen (1989a), "Savings Promotion, Investment Promotion, and International Competitiveness," in R. Feenstra (ed.), **Trade Policies for International Competitiveness**, University of Chicago Press, Chicago.

Grais, W., J. de Melo and S. Urata (1986), "A General Equilibrium Estimation of the Effects of Reductions in Tariffs and Quantitative Restrictions in Turkey in 1978," in T. Srinivasan and J. Whalley, eds.

Gunasekera, D.B.H., and R. Typers (1988), "Imperfect Competition and Returns to Scale in a Newly Industrializing Economy: A General Equilibrium Analysis of Korean Trade Policy," processed, National Centre for Development Studies, Australian National University, Canberra.

Hanson, K., S. Robinson, and S. Tokarick (1989), "United States Adjustment in the 1990's: A CGE Analysis of Alternative Trade Strategies," Working Paper No. 510, Department of Agricultural and Resource Economics, University of California, Berkeley.

Harris, R.G. (1984), "Applied General Equilibrium Analysis of Small Open Economies with Scale Economies and Imperfect Competition," **American Economic Review**, Vol. 74, No. 5, pp. 1017-1032.

Harris, R.G. (1988), "Alternative Solution Procedures in Applied General Equilibrium Analysis," Working Paper 53, OECD Department of Economics and Statistics, Paris.

Harris, R.G., and D. Cox (1984), **Trade, Industrial Policy, and Canadian Manufacturing**, Ontario Economic Council, Toronto.

Henderson, Y.K. (1989), "Tax Modelling in the Last Five Years," paper presented at the NBER Workshop on Applied General Equilibrium Analysis, San Diego, September 5-7.

Hertel, T.W. (1989), "Applied General Equilibrium Analysis of Agricultural Policies," paper presented at the NBER Workshop on Applied General Equilibrium Analysis, San Diego, September.

Hertel, T.W., R.L. Thompson, and M.E. Tsigas, 1989, "Economywide Effects of Unilateral Trade and Policy Liberalization in U.S. Agriculture," in A.B. Stoeckel, D. Vincent, and S. Cuthbertson (eds.), **Macroeconomic Consequences of Farm Support Policies**, Duke University Press, Durham, Forthcoming.

Horridge, M., and D. Pearce, 1988, "Modelling the Effects on Australia of Interventions in World Agricultural Trade," IMPACT Preliminary Working Paper No. OP-65, University of Melbourne.

Johansen, L. (1960), **A Multi-Sectoral Study of Economic Growth**, North-Holland, Amsterdam.

Jones, R.W. (1965), "The Structure of a Simple General Equilibrium Model," **Journal of Political Economy**, 73, pp.557-572.

Jorgenson, D.W., and P. Wilcoxen (1989), "Environmental Regulation and U.S. Economic Growth," paper presented at the NBER Workshop on Applied General Equilibrium Analysis, San Diego, September 5-7.

Kehoe, T.J., A. Manresa, P Javier Noyola, C. Polo, F. Sancho, and J. Serra-Puche (1986), "A General Equilibrium Analysis of Indirect Tax Reform in Spain," Discussion Paper W.P.66.86. Departament de Teoria Economica i Econometria, Universitat Autonoma de Barcelona, Barcelona.

Kehoe, T.J., and J. Serra-Puche (1983), "A Computational General Equilibrium Model with Endogenous Unemployment: An Analysis of the 1980 Fiscal Reform in Mexico," **Journal of Public Economics**, 22:1, pp.1-26.

Lee, H., and D. Roland-Holst (1992) "International Trade and the Transfer of Environmental Costs and Benefits," in J.F. Francois and K.A. Reinert (eds.), **Applied Trade Policy Modeling**, Cambridge University Press, London, Forthcoming.

Loo, T., and E. Tower, 1988, "Agricultural Protectionism and the Less Developed Countries," in A.B. Stoeckel, D. Vincent, and S. Cuthbertson (eds.), **Macroeconomic Consequences of Farm Support Policies**, Duke University Press, Durham, Forthcoming.

McCarthy, F.D., and L. Taylor (1980), "Macro-Food Policy Planning: A General Equilibrium Model of Pakistan," **Review of Economics an Statistics**, 62:1, pp.107-121.

Melo, J. de and S. Robinson (1989), "Product Differentiation and the Treatment of Foreign Trade in Calibrated General Equilibrium Models of Small Economies," **Journal of International Economics**, 27, pp.47-67.

Melo, J. de (1988), "Calibrated General Equilibrium Models for Trade Policy Analysis in Developing Countries: A Survey," **Journal of Policy Modelling**,10:3, Fall.

Melo, J. de, and D. Roland-Holst (1992) "Structural Adjustment to U.S. Import Restraints When Factor Services are Tradable," in J. Shoven and J. Whalley (eds.), **Advances in Applied General Equilibrium Analysis**, Cambridge University Press, Forthcoming.

Melo, J. de, and D. Roland-Holst (1992) "Tariffs and Export Subsidies When Domestic Markets are Oligopolistic," in J. Mercenier and T.N. Srinivasan (eds.), **Applied General Equilibrium and Economic Development**, University of Michigan Press, Forthcoming.

Melo, J. de, and D. Tarr (1989), **A General Equilibrium Analysis of U.S. Foreign Trade Policy**, MIT Press, Cambridge.

Melo, J. de, and D.W. Roland-Holst (1989b), "International Capital Mobility and the Cost of U.S. Import Restraints," Working Paper, U.S. International Trade Commission, Washington, D.C., October.

Melo, M. de (1979), "Agricultural Policies and Development: A Socio-economic Investigation Applied to Sri Lanka," **Journal of Policy Modeling**, 1:2, pp.217-234.

Melo, M. de (1980), "The Impact of Trade Policies on Income Distribution in a Planning Model of Colombia," **Journal of Policy Modeling**, 2:1, pp.81-100.

Michel, G., and M. Noel (1984), "The Ivorian Economy and Alternative Trade Regimes," in W. Zartman and C. Delgado (eds.), **The Political Economy of the Ivory Coast**, Praeger, New York.

Mitra, P., and S. Tendulkar (1986), "Coping with Exogenous Internal and External Shocks: India, 1973-74 to 1983-84," processed, Country Policy Department, World Bank, Washington, D.C.

Newbery, A., and N. Stern (1988), **The Theory of Taxation for Developing Countries**, Oxford University Press, New York.

Polo, C., D. Roland-Holst, and F. Sancho (1992) "Analysis de la Influencia Economica en un Modelo Multisectorial," **Revista de Economia Publica**, Forthcoming.

Polo, C., D. Roland-Holst, and F. Sancho (1992) "Decomposicion de Multiplicadores en un Modelo Multisectorial: Una Aplicacion al Case Español," **Investigaciones Economicas**, Forthcoming.

Polo, C., D.W. Roland-Holst, and F. Sancho (1989), **Análisis estructural de la economía española**, Universitat Autonoma de Barcelona, Barcelona, forthcoming.

Reinert, K.A., and D. Roland-Holst (1992) "Social Accounting Matrices for Trade Policy Modeling," with K.A. Reinert, in J.F. Francois and K.A. Reinert (eds.), **Applied Trade Policy Modeling**, Cambridge University Press, London, Forthcoming.

Reinert, K.A., D. Roland-Holst, and C.R. Shiells (1992) "North American Trade Liberalization and the Role of Nontariff Barriers," in J.F. Francois and C.R. Shiells (eds.), **Modeling Trade Policy: Applied General Equilibrium Analysis of the North American Free Trade Agreement**, Cambridge University Press, London, Forthcoming.

Reinert, Kenneth, and Clinton R. Schiells, 1991, "Trade Substitution Elasticities for Analysis of a North American Free Trade Area," U.S. International Trade Commission, Washington, D.C., January.

Reinert, Kenneth, and David Roland-Holst, 1990a, "General Equilibrium Estimates of the Cost of U.S. Import Protection," U.S. International Trade Commission, Washington, D.C., June, 1990, 135pp.

Reinert, Kenneth, and David Roland-Holst, 1990b, "An Empirical Study of Export Responsiveness in U.S. Mining and Manufacturing Industries," U.S. International Trade Commission, Washington, D.C., Forthcoming Fall, 1990.

Reinert, Kenneth, and David Roland-Holst, 1991a, "A Detailed Social Accounting Matrix for the United States," Paper Presented at the International Symposium on a DAM for Europe, University of Valencia, Spain, September, 1990.

Reinert, Kenneth, and David Roland-Holst, 1991b, "Disaggregated Armington Estimates for U.S. Mining and Manufacturing Sectors," **Journal of Policy Modeling**, Forthcoming.

Robinson, S. (1988), "Multisectoral Models for Developing Countries: A Survey," in H.B. Chenery and T.N. Srinivasan (eds.), **Handbook of Development Economics**, North-Holland, Amsterdam.

Roe, A.R., and S. Pal (1986), "Kenya's Adjustment to the Oil Crises, 1972-82," processed, Country Policy Department, World Bank, Washington, D.C.

Roland-Holst, D.W. (1989), "Comments on Goulder and Eichengreen: Savings Promotion, Investment Promotion, and International Competitiveness," in R. Feenstra (ed.), **Trade Policies for International Competitiveness**, University of Chicago Press, Chicago.

Roland-Holst, D.W., and E. Sadoulet (1989), "Trade Policy, Protection, and Resource Allocation in Ecuador," processed, Department of Agricultural and Resource Economics, University of California, Berkeley.

Roland-Holst, D.W., and E. Thorbecke (1988), "The Debt Problem, Stabilization and Structural Adjustment and their Socioeconomic Impact: The Case of Indonesia," paper presented at the Annual Meetings of the American Economic Association, New York.

S.J. Burky and G. Perry (1997), "The Long March. A reform agenda for Latin America and the Caribbean in the next decade", World Bank, Washington, D.C.

Sadoulet, E. (1986), "A CGE Model of Morroco," in Mateus et al, **Structural Adjustment Policies: The Case of Morroco**, World Bank, Washington, D.C.

Seth, R. (1986), "A Colombia: An Assessment of Alternative Borrowing Strategies," processed, Country Policy Department, World Bank, Washington, D.C.

Shoven, J. and J. Whalley (1984), "Applied General Equilibrium Models of Taxation and International Trade," **Journal of Economic Literature**, 22(3), pp. 1007-1051.

Stern, R., J. Francis and B. Schumacher (1976), **Price Elasticities in International Trade: An Annotated Bibliography**, Trade Policy Research Centre, Macmillan Press, London.

Stone, R.F. (1981), **Lectures on Social Accounting**, Chapman and Hall, London.

Taylor, L., E. Bacha, E. Cardoso, and F.J. Lysy (1980), **Models of Growth and Distribution in Brazil**, Oxford University Press, Oxford.

Valdes, A., and J. Zietz, 1980, **Agricultural Protection in OECD Countries: Its Costs to Less Developed Countries**, Research Report 21, IFPRI, Washington, D.C.

Vincent, D.P. (1989), "Domestic Effects of Agricultural Policies in Asian Countries with Special Reference to Korea," in A.B. Stoeckel, D. Vincent, and S. Cuthbertson (eds.), **Macroeconomic Consequences of Farm Support Policies**, Duke University Press, Durham, Forthcoming.

Whalley, J. (1985), **Trade Liberalization Among Major World Trading Areas**, Cambridge, Mass.: MIT Press.

Whalley, J. (1989a), **The Uruguay Round and Beyond**, Macmillan, London.

Whalley, J., and R.M. Wigle (1988), "Endogenous Participation in Agricultural Support Programs and Ad Valorem Equivalent Modeling," NBER Working Paper No. 2583, Cambridge.

Wigle, R.M. (1988), "General Equilibrium Evaluation of Canada-U.S. Trade Liberalization in a Global Context," **Canadian Journal of Economics**, 21:3, pp.539-564.

7 Figures

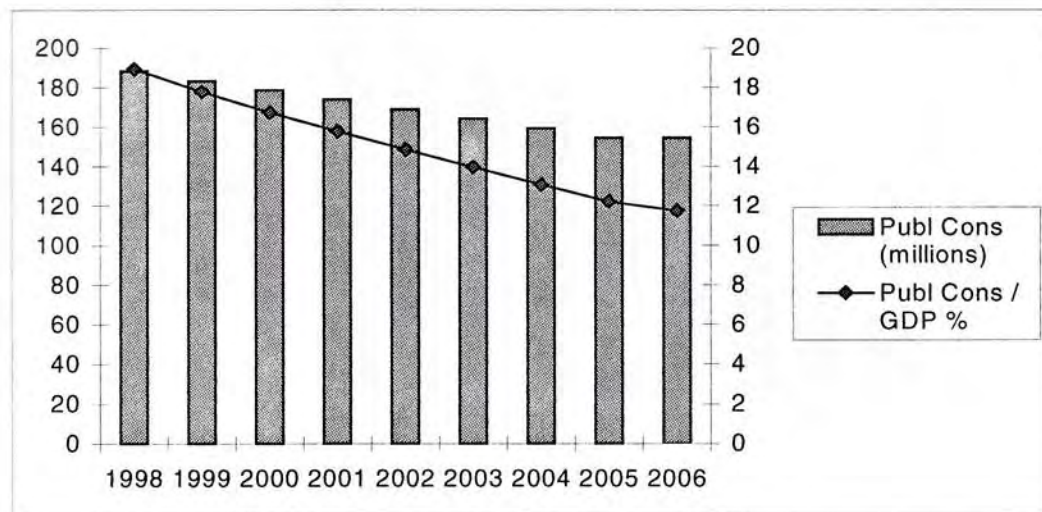*Figure 6-1: Government consumption reduction policy*

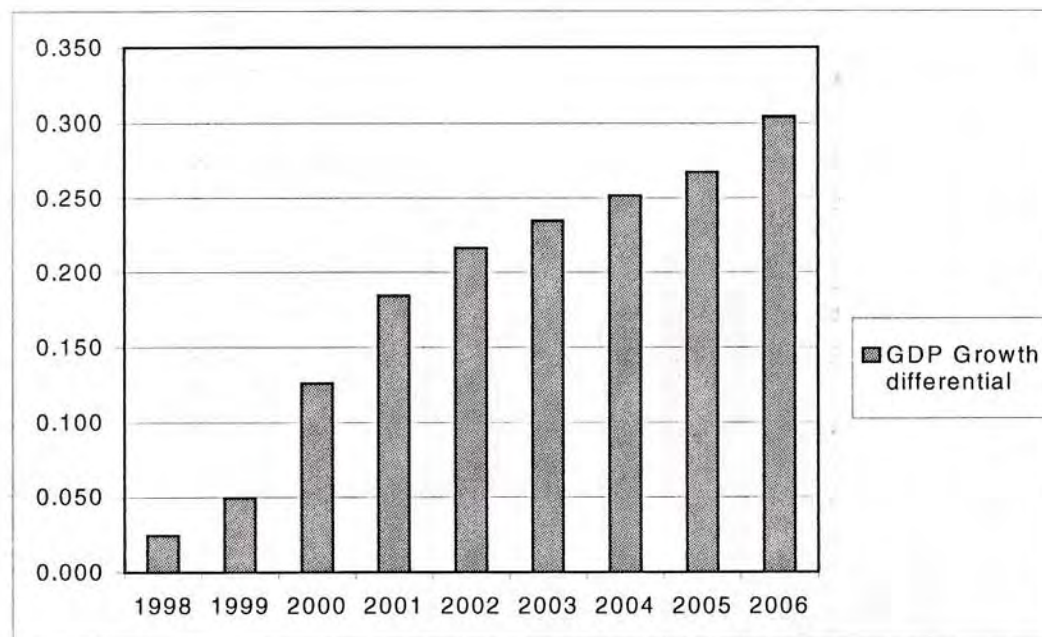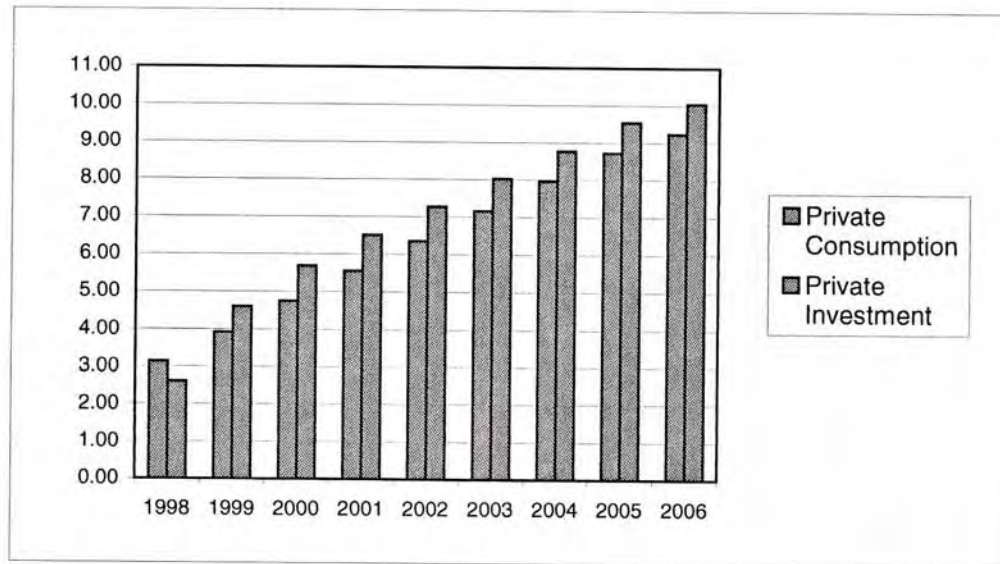*Figure 6-2: GDP growth rates differentials*

Figure 6-3: Private Investment and Consumption differentials

Figure 6-4: Labour market effects